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Activities to Learn the Proportion Concept Using Technology

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Abstract
The research reported in this paper has a bearing on qualitative proportional thinking, for which technology is used; specifically an interactive computational program which supports the construction of the concept of proportion in a qualitative way is developed. The indicators of this concept are constructed and the ones of the development of qualitative proportional thinking, the design of didactic activities and their insertion in the interactive computational program is shown, as well as its implementation, to finish with the analysis and discussion of results.

Keywords: Activities, Computer aided instruction, Elementary School, Mathematical Educational, Software.

1. Introduction

The development of proportional thinking is important since basic educational levels, as from it depend that children be able to comprehend and face everyday situations which are linked with the concept of proportion. At the same time, as it is established by Ruiz [1], in order for the student at basic level to be able to assign sense and meaning to proportion it is fundamental to develop his proportional thinking, both the qualitative and the quantitative. That is to say, for the development of proportional thinking it is required, among others that the subject constructs the concept of proportion, and in order for him to be able to construct this concept it is required to have proportional thinking. In other words, there exists a bidirectional relationship between the mathematical concept of proportion and proportional thinking.

In order for a child to identify the proportional, in accordance with Piaget (1978) [2], this should be made starting from the concrete to reach the abstract. Similarly, the construction of this mathematical concept is represented both at a qualitative level and at a quantitative one, which determines the proportional thoughts both qualitative and quantitative, respectively.
From another perspective, nowadays, within the educational contexts there exists the possibility of designing and developing technological materials of an interactive type. Whatever classroom experience can use electronic technology as mediation way, but it is necessary to value dimensions such as: objectives to be reached, organization of the themes

which enable the development of the preferred contents to be included, learning activities and the evaluation both of the learning and the overall process. [3]- [7]

On the other hand, the increment of technological materials is amazing; however, didactic strategies designed with theoretical elements are not always included. In the research being shown, in this paper, the educational theory named *Mathematics within the Context of Sciences* is used, by means of which the mathematical themes and concepts should be treated is such a way that they are linked to the environment, to everyday activities, to labor and professional competencies, as well as to the other sciences studied by the pupil.

From what has been described, a research has been dealt with whose problem is the construction of the concept of proportion and the development of proportional thinking both qualitative and quantitative, by means of didactic activities with electronic technology, in sixth grade children from elementary education. Since the research is so vast, this presentation only reports one didactic activity for the development of qualitative proportional thinking, as well as its implementation and analysis, which constitutes a partial research.

The objective of the partial research which is reported is to design didactic activities for the construction of the concept or proportion (in a qualitative way) and the development of qualitative proportional thinking, by means of an interactive computational program.

The supposition of research emerges from the fact that upon constructing or reconstructing the concept of proportion, there will be, to some extent, a bearing on the development of proportional thinking, for this report, on the qualitative proportional thinking.

2. Methodology

2.1 Review Stag

The design of the activities for the development of qualitative proportional thinking, by means of an interactive computational program, uses a methodology provided in the following three steps:

1. To determine the indicators associated with the construction of the concept of proportion (in a qualitative way), as well as the indicators associated with the development of qualitative proportional thinking (by means of the mathematical concept).
2. To design the didactic activities on the concept of proportion and their insertion in the interactive computational program.
3. To implement the interactive computational program and, to analyze and to discuss the results.

2.2 Theoretical Foundation

The theory of *Mathematics within the Context of Sciences* establishes that mathematics should be presented to the student, at any educational level, through contextualized events, [8]. This theory conceives the learning and teaching process as a system where the five phases of the theory intervene: curricular, cognitive, didactic, epistemological and educational. As a theory, in each one of its phases a methodology with theoretical foundation is included, in accordance with the paradigms upon which it is based, where the steps for the curricular design are guided, the didactics to be followed is described, among others.

2.3 Research Development

2.3.1 Determination of Indicators

The authors who are mentioned in this section also belong in the section of theoretical foundation, it is just that in order not to duplicate the information they have only been located in the present section. It is through them that the indicators for the development of the concept of proportion are identified, as well as the indicators for the development of qualitative proportional thinking. It is worth mentioning that the identified indicators have been highlighted in bold type.

As an antecedent of what has just been mentioned stands the research by Ruiz [9], who designed and applied a teaching proposal dealing with the concept of proportion and who found out different difficulties presented by students in the last year of elementary education, among them we find the two following: 1. Qualitative thinking of elementary school students revolving around proportionality is barely developed. 2. Students showed confusion when establishing proportions in an intuitive way and explicitly in geometric figures, since they were not able to establish proportions when they compare it in figures of similar length with the width of another.

Piaget (1978) [2] argues that between 11 and 12 years of age, it is possible to observe in the subject the presence of the notion of proportion in different fields, such as: spatial proportions (similar figures), the relationships between weights and lengths of the arms of the scales, the probabilities, etc. Piaget also mentions that through his experiments he points out that the child acquires the qualitative identity before quantitative conservation and makes a distinction between qualitative comparisons and the true quantification. In fact, for Piaget [2] the notion of proportion always starts in a qualitative and logical way, before being quantitatively structured. He stresses that I order for the student to develop his *qualitative proportional thinking* it is necessary to start from the notions of ***enlargement and reduction (1)***, following the idea of a photocopy or a scale drawing, assuming that the student at a really early age manages to recognize what is proportional using perception and observation. One way to express his *qualitative thinking* is to use linguistic expressions such as “greater than…” and “lesser than…”, that is to say, ***using verbal categories (2)***

In accordance with Piaget and Inhelder [10], after the student develops the perceptual part (qualitative proportional thinking) an ordering when making comparisons appears (which is located in the *shift from qualitative proportional thinking to quantitative proportional thinking*), this can be verified when the student compares figures superimposing them taking as origin the indicator ***to compare (3)***. In this respect, Piaget points out that in this *shift from the qualitative to the quantitative* the student can build a figure enlarging it or reducing it, thus constituting the indicators ***to enlarge and to reduce (5)***. Later on, the student uses the measurement when making comparisons, firstly confronting parts of the object and superimposing one figure over another and then using a measuring instrument, conventional or not. Thus, ***measuring with instruments (6)*** represents another indicator, allowing the development of his *quantitative proportional thinking.*

It is important that the student when developing his *qualitative proportional thinking* manages to ***use the rule of three (7)*** assigning sense to this and not merely in a mechanical way , with which one more indicator is defined. [1]

In terms of Freudenthal (1983) [11], in order to establish proportions, *both intuitively* (that is, qualitatively) *and explicitly* (that is, quantitatively), the comparisons are expressed in two modalities: direct and indirect. The direct modality of comparing is when and object is ***superimposed*** over another object, which defines the indicator ***to compare directly (8)***, whereas the indirect one is when there are two objects and an ***instrument*** to compare them, like the use of a ruler or simply by counting obtaining as a result the indicator ***compare indirectly (9).*** The child is able to compare two objects indirectly and is able to do it in a qualitative and/or quantitative way.
It is important to mention that authors such as Piaget [2] and Streefland (1991) [12] mention in a natural way qualitative proportional thinking is first developed, through the perception of the empirical. On the other hand, it has been demonstrated that in the educational practice the use of algorithm is given priority, thus students developing a quantitative proportional thinking in a mechanic way, when in many occasions they have not developed qualitative

proportional thinking. Thus, the qualitative-quantitative sequence is not always present in students.

Freudenthal [11] points out that the comprehension, in an intuitive way, (i.e. qualitative), of proportion can be guided and delved in by the use of visualization and the latter can be illustrated using detailed constructions, where the drawings are differentiated and show what points correspond among themselves in the original and in the image. Freudenthal also suggests that upon working on the proportion of lengths flat figures are used as means of representation, for its expressiveness is more global, in the sense that they make it easier for the student the qualitative and quantitative comprehension between magnitudes by means of visual perception. From this perspective of mathematics, it is also important that the student manages to express *proportion* ***as a fraction (10)*** which constitutes one more indicator. Moreover, for Freudenthal [11], it is precise that in teaching it is precise to take into account both ***internal proportions (11)*** and ***external proportions (12)***, defining the former as the relationships which are established between different values of the same magnitude, and the latter as relationships between values of different magnitudes; both proportions express two indicators to be taken into account.

Resorting to what has been mentioned by researchers in the previous paragraphs, a table in which the indicators referred to proportion as well as to qualitative proportional thinking is shown next; the didactic actions associated to such indicators are established as well.

Table 1: Indicators and their didactic actions

Objects of study	**Objectives**	**Indicators**	**Didactic actions**
The mathe-matical concept of propor-tion	To establish proportions in a qualitative way	Compares directly (8) Compares indirectly(9)	♦ To superimpose figures ♦ To use a measuring instrument
	To establish proportions in quantitative way	Compares indirectly (9) Uses internal and external propor-tions (11, 12) Expresses proportion as a fraction (10)	♦ To use a measuring instrument ♦ To use a table relating data and writing the proportion as a fractions
The development of qualitative proportional thinking	To contribute to the development of qualitative proportional thinking	Enlarges and reduces (1) Uses verbal categories such as “greater than” or “lesser tan” (2)	♦ To select reduced figures or enlarged ones by means of ♦ To use linguistic expressions
	To contribute to the shift from qualitative proportional thinking to quantitative.	Compares (3) Counts (4) Enlarges and reduces figures (5)	♦ To superimpose figures ♦ To count the sides of squares in one grid ♦ To draw enlarged or reduced figures in a grid

It can be observed in table 1 the diverse didactic actions which students should work upon, and which they can carry out in the interactive computational program. These didactic actions are expressed in a generic way like: To superimpose figures, To use verbal categories* To use measuring instruments, To use tables*, To select figures, To draw figures in a grid, To count the squares in a grid.

The actions marked with an asterisk, require of a recorder, which is external to the interactive computational program. When the student carries out these didactic actions it is necessary to use the recorder to have evidence. The verbal categories are recorded in the audio, as well as the comments by the students. The didactic actions “to use tables”, allows the student to relate date from the same column, between two columns or that the table can be filled out by him. For the case of relating data, the audio allows to identify this indicator.

2.3.1. 1st The Link between Didactic Actions and Computational Actions

The computational program was developed in order for the student to be able to carry out the didactic actions. For this research the computational program which has been developed allows to incorporate actions such as to drag figures, to access the figures of a box, to use a virtual pencil as though it were real, to make use of a grid to count the squares or to draw upon it, to make measurements with instruments such as a virtual ruler, to use tables to be filled out; al the actions afore mentioned are named ***computational actions.***

In table 2 the link between didactic actions and computational actions, necessary for the constructions of the concept of proportion, as well as the concept of qualitative proportional thinking is established. That is, computational actions are the ones which allow that didactic actions associated with each indicator of the construction of the objects of study can be carried out.

Table 2: Link between didactic actions and computational actions

Didactic actions	**Computational actions**
To superimpose figures	To drag figures
To use measuring instruments	To use a virtual ruler
To use tables	To fill out a table
To select figures	To access figures
To draw figures in a grid	To use a virtual pencil
To count the squares in a grid	To use a grid

2.3.2 Design of Didactic Activities and Insertion in the Interactive Computational Program

In this section are first described the pedagogical elements from the interactive computational program which was overtly designed for this project. Later one the didactic activity is shown, specifying its purpose, which in founded upon the theoretical elements previously shown; the contextualized event which identifies it, in accordance with the theory of Mathematics within the Context of Sciences; the didactic actions which can be used by the student when dealing with the activity and, the computational action associated with this event; to finish with the insertion in the interactive computational program.

We present next the pedagogical elements from the interactive computational program. The pedagogical elements form the computational program allow the representation and visualization of instructions, the activities to be carried out, the help and options which the user can resort to, etc., in general the elements which are used in the learning process.

Figure 1: Screens from the interactive computational program

Figure 1: Screen from the interactive computational program

Figure 1: Screens from the interactive computational program

In accordance with Mathematics within the Context of Sciences, a familiar environment as well as metaphors of the real world adapted to sixth grade elementary students are presented, see figure 1a. The program is user friendly as it allows the user to log out, log in and send. For instance, you hold a click on the door to log out, see same figure 1. A visual style is presented by means of font types, buttons and general aspects focused on elementary school children. Brevity in the texts is used, incorporating sounds and graphics to substitute possible content in the text. The type, style and size on fonts are legible. Messages are simple and concise, so as to avoid the user being confused.

The user is given tolerance of three attempts to correctly solve the activities and when he carries out an activity correctly, he is warned as a motivating means, see figure 2. In the interactive computational program the student can look up his advance, see figure 3.
When the student does not accomplish to carry out the activity correctly, then, animations are used to illustrate the concepts by means of examples and thus the association between concepts is immediately generated. Next we present the activity and its insertion in the interactive computational program.

2.3.2. 1st Purpose of the Activity

The activity pursuits the construction of the concept of proportion and in consequence the development of qualitative proportional thinking. We start from what has been pointed out by Piaget [2], Streefland [12] and Ruiz [8], on the fact that early teaching of proportion should start from qualitative levels of recognition of them, for this reason, the activity does not require of the use of quantities for its solution.

2.3.2.2st Contextualized Event of the Activity

"Ricardo is ten years old, he went to Veracruz and at the harbor he saw a ship which liked him very much, the child drew it in a sheet of pager. When he came back to school he showed it to his teacher. She asked for four versions of the ship at the photocopier, some of them enlarged and others reduced, in different sizes. After that she took the photocopies to her students and asked them to choose the figure of the ship which was enlarged twice the size. Help them to find the ship which is enlarged".

2.3.2.3st Didactic and Computational Actions of the Activity

In order to solve this activity the student can make use of the following didactic action: *to select the reduced figure* or the enlarged one to the given one. This corresponds to indicator (1): *to enlarge and reduce*, from the development of qualitative proportional thinking and is associated with the computational action of *accessing figures*.

The student can also make use of the didactic action of *superimposing figures*, this didactic action corresponds to indicator (8) of *directly comparing* when we aim at establishing proportions in a qualitative way (intuitive), similarly, it corresponds to indicator (3) of *comparing* the development of the shift from qualitative proportional thinking to the quantitative one.

In order to superimpose the figures, the student has the option to drag, with the mouse, whichever of the four figures to superimpose them over the original one and revise, by the use of visualization, whether the figures is enlarged or reduced in all the sides in the same quantity, the computational action involved corresponds to *drag figures*.
The recorder is turned on all the time in order to have the comments by students and their *linguistic expressions* which allow us to identify the qualitative proportional thinking through indicator (2) of *using verbal categories*.

If, not even in a second attempt is the student able to select the correct figure, it means that his qualitative proportional thinking is scarcely developed; however, as it has been mentioned, some students develop more of a kind of thinking than another, that is why they are given the option to move to didactic actions from quantitative proportional thinking. Then, there is the didactic action of *using a measuring instrument*, associated with indicator (9) of *comparing indirectly*, which corresponds to the construction of the concept of proportion both in a qualitative, and in a quantitative way, it will depend on whether the proportion is established in a numerical way (quantitative) or it is only verbalized (qualitative). At the same time, this didactic action is associated with indicator (6) of *measuring with instruments* from quantitative proportional thinking. The corresponding computational action is a *virtual ruler*, which allows the student to measure each side of the original figure, as well as each side of the figures that appear to select the one which is enlarged. It is worth mentioning that this interactive computational program randomly generates the figures (it is not always a ship), as well as their enlargement or reduction in different scales (which are not always expressed in whole numbers), with the purpose that it does not become a mechanic activity and that it is not memorized. In general the computational program randomly generates the data, the figures, the contexts and the tables.

2.3.2.4st Insertion of the Activity in the Interactive Computational Program

In a first screen the figure of the ship and 4 figures which are similar are shown, but with Little differences among them, see figure 1b. For instance, they can be twice the size, thrice the size, reduced to half or to one third of the original size and the child is asked to choose from the 4 figures the one which is enlarged to twice the size of the original.

When the child has selected one of the figures, it is because he has visualized the enlargement (1) or has dragged the figures in order to superimpose them and then be able to compare them directly (8) and define which one is enlarged twice the size. After selecting the figure, the computational program analyzes the choice made and following it sends an answer with the result of the analysis: a) If it is correct, another activity is shown to the child. B) If the selection is incorrect, the program sends a written message saying “the selection is not correct” and it asks if the child wants to have another attempt. If the child answers affirmatively, then, the same activity with the same data appears, but now with an auxiliary tool which is a virtual ruler, as it has been mentioned, which enables him to measure and make comparisons (indirect 9,6). If even with this support of the virtual ruler the selection is again incorrect, then, the interactive computational program presents another version of the same activity, which is randomly generated with other figures and other data, in order for the child to try again.

If after these attempts the student has not been able to select the correct figure, then, he is presented with one simulation of the activity, with other figures and other date, in such a way that the simulator superimposes figures and drags the virtual ruler, appearing the data of the measurements of the figures, in this way it can be by the actions performed by the simulator which figure is enlarged or reduced, in accordance with what is being asked. This allows the child to associate these actions with the ones he should have made in the attempts he carried out.

3. Implementation, Analysis and Discussion of Results

For the implementation we had a group of 29 students from sixth grade of elementary education in Mexico, specifically from a public school in Mexico City. The ages of the students ranged fluctuated between eleven and twelve years old. Six sessions were devoted for the work with students, each one lasting two hours.

The analysis and discussion of results are carried out in relation to the indicators of the construction of the concept of proportion (in a qualitative way) and to the ones of the development of qualitative proportional thinking.

Seven out of twenty-nine students are identified who chose the enlarged figure (Ind. 1), by means of visualization, though; it is worth mentioning that two of them managed to do that in the second attempt. However, in general it can be said that 24% of students, by means of visualization are able to identify the proportions and select the correct figures, that is, they have developed their qualitative proportional thinking. This can be observed through the recordings of the verbal categories (Ind.2) by students, such as “this is greater than the other”, “it appears that this is twice the size”, which were expressed, by six out of the seven students, during this activity.

The students who have not developed this qualitative proportional thinking are unable to visually identify the correct figure and they need to resort to direct comparisons and indirect ones. In the direct comparison they make use of the didactic action of superimposing figures, whereas in the indirect comparison, the didactic action is the use of a measuring instrument.
During the activity, eight out of twenty nine students drag figures thus fulfilling the indicator of comparing directly (Ind.8), which provides evidence that they can establish proportions in a qualitative way. Note that the action of superimposing figures also determines the indicator of comparing (Ind.3), which favors the shift from qualitative proportional thinking to the quantitative one. Thus, 28% of the students start to develop their qualitative proportional thinking and shifting to quantitative proportional thinking. Moreover, comments by three out of the eight students are recorded, such as: “this side is twice the size of the other” or “this side fits twice in the other”, denoting the use of internal proportions (Ind.11) see table 1, which provides evidence of an incipient use of proportions in a quantitative way.

The remaining fourteen out of twenty-nine students used the two attempts which are given by the system to make a selection by means of visualization or the dragging of figures, without success. Moreover, in the audio recordings they registered linguistic expressions that show their difficulty to make the selection such as “the tree figures which are greater resemble each other”. It is possible to say that these students have not managed to develop their qualitative proportional thinking.

Upon not being successful, the interactive computational program provides them with a virtual ruler. Only six out of the fourteen students measured (Ind. 6) the sides of the figures and compared them using this instrument identifying the correct figure, with which the indicator of comparing indirectly (Ind.9) is verified. Upon being measuring, the students obtain numerical values which compare sides to corresponding sides, establishing internal proportions (Ind. 11); which gives rise to establish ratios in an explicit way; six out of the fourteen students when carrying out this actions show signs of the development of their quantitative proportional thinking.

The remaining eight students out of the fourteen who used the virtual ruler, only measured one of the sides and selected the figures which had twice the size of the homologous side, but they did not notice that they had to measure all the sides of the figure to compare them to the figure they selected. These errors led them to the unsuccessful selection of the correct figure, which shows us that these eight students have not developed their qualitative proportional thinking either.

4. Conclusions

The different computational actions, dragging with the mouse, and using a virtual ruler, have a bearing on the didactic actions of superimposing one figure over the other to make the comparison or using a measuring instrument. All that depended on the kind of proportional thinking which students had previously developed either the qualitative or the quantitative one.

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